ESAME MATEMATICA E STATISTICA 27/02/24

$\boxed{\text{ESERCIZIO 1}}$

a. $f(x) = \dfrac{x^2+6x+6}{x^2+2x+2}$ C.E. $x^2+2x+2 \neq 0$

$\Delta = 4-8 = -4$

$\Downarrow$

NO SOLUZIONI $\rightarrow$ $D = \mathbb{R}$

- INTERSEZIONE ASSE y : $f(0) = \frac{6}{2} = 3$ $A = (0,3)$

- STUDIO DEL SEGNO E INTERSEZIONI ASSE x

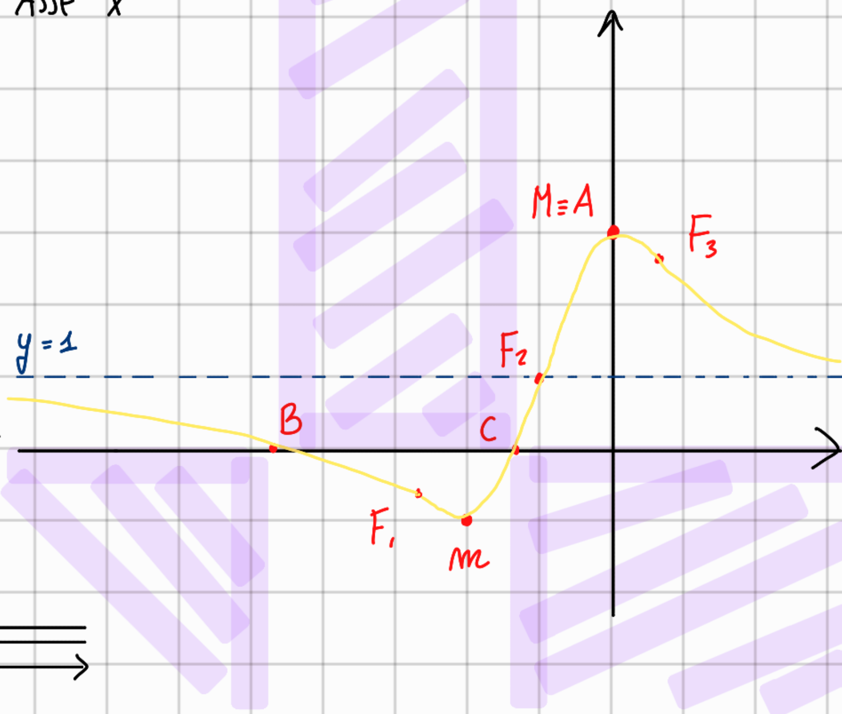
$$\frac{x^2+6x+6}{x^2+2x+2} \geq 0$$

- $x^2+6x+6 \geq 0$

$\Delta = 36-24 = 12$

$x_{12} = \dfrac{-6 \pm \sqrt{12}}{2}$ $\nearrow x_1 = \dfrac{-6-2\sqrt{3}}{2} = -3-\sqrt{3} \simeq -4.73$

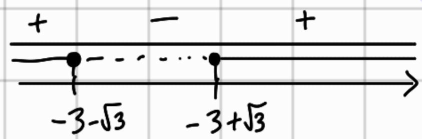
$\searrow x_2 = \dfrac{-6+2\sqrt{3}}{2} = -3+\sqrt{3} \simeq -1.27$



- $x^2+2x+2 > 0$

$\Delta < 0$

$\mathbb{R}$



$B = (-3-\sqrt{3}, 0)$, $C = (-3+\sqrt{3}, 0)$

- $f(x)$ POSITIVA $x \in (-\infty, -3-\sqrt{3}) \cup (-3+\sqrt{3}, +\infty)$
- $f(x)$ NEGATIVA $x \in (-3-\sqrt{3}, -3+\sqrt{3})$
- $f(x)$ NULLA $x = -3-\sqrt{3}$, $x = -3+\sqrt{3}$

- LIMITI AGLI ESTREMI DEL DOMINIO

$\lim_{x\to-\infty} \dfrac{1x^2+6x+6}{1x^2+2x+2} = \dfrac{1}{1} = 1$ PER CONFRONTO DI INFINITI

$\lim_{x\to+\infty} \dfrac{x^2+6x+6}{x^2+2x+2} = 1$ PER CONFRONTO DI INFINITI

ASINTOTO ORIZZONTALE COMPLETO $\boxed{y=1}$

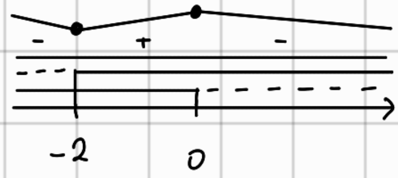
- DERIVATA PRIMA

$$f'(x) = \frac{(2x+6)(x^2+2x+2)-(2x+2)(x^2+6x+6)}{(x^2+2x+2)^2} = \frac{\cancel{2x^3}+6x^2+4x^2+\cancel{12x}+4x+\cancel{12} \ -\cancel{2x^3}-2x^2-12x^2-\cancel{12x}-12x-\cancel{12}}{(x^2+2x+2)^2} = \dots$$

$$\dots = \frac{-4x^2-8x}{(x^2+2x+2)^2} = \frac{-4x(x+2)}{(x^2+2x+2)^2}$$

$f'(x) \geq 0$

$-4x \geq 0$	$x+2 \geq 0$	$(x^2+2x+2)^2 > 0$
$x \leq 0$	$x \geq -2$	$\mathbb{R}$



f è CRESCENTE $x \in (-2, 0)$

f è DECRESCENTE $x \in (-\infty, -2) \cup (0, +\infty)$

f è STAZIONARIA $x = -2,\ x = 0$

$$f(-2) = \frac{4-12+6}{4-4+2} = \frac{-2}{2} = -1$$

$m = (-2, -1)$ P.TO DI MINIMO

$M = (0, 3) \equiv A$ P.TO DI MASSIMO

- DERIVATA SECONDA

$$(8x^2+16x)(2x+2)$$

$$f''(x) = \frac{(-8x-8)(x^2+2x+2)^{\cancel{2}} - (-4x^2-8x)2\cancel{(x^2+2x+2)}(2x+2)}{(x^2+2x+2)^{\cancel{4}\,3}} = \frac{-8x^3-8x^2\cancel{-16x^2}\cancel{-16x}\cancel{-16x}-16+16x^3+32x^2\cancel{+16x^2}\cancel{+32x}}{(x^2+2x+2)^3}$$

$$= \frac{8x^3+24x^2-16}{(x^2+2x+2)^3} = \frac{8(x^3+3x^2-2)}{(x^2+2x+2)^3} = \boxed{\frac{8(x-1)(x^2+2x-2)}{(x^2+2x+2)^3}}$$

$$\begin{array}{c|ccc|c} & 1 & 3 & 0 & -2 \\ -1 & & -1 & -2 & 2 \\ \hline & 1 & 2 & -2 & / \end{array}$$

$8(x-1) \geq 0$	$x^2+2x-2 \geq 0$	$(x^2+2x+2)^3 > 0$
$x \geq 1$	$\Delta = 4+8 = 12$	$x^2+2x+2 > 0$
		$x \in \mathbb{R}$

$$x_{12} = \frac{-2 \pm 2\sqrt{3}}{2} \begin{cases} x_1 = -1-\sqrt{3} \\ x_2 = -1+\sqrt{3} \end{cases}$$

$x < -1-\sqrt{3} \ \vee \ x > -1+\sqrt{3}$

f CONCAVA $x \in (-\infty, -1-\sqrt{3}) \cup (-1, -1+\sqrt{3})$

f CONVESSA $x \in (-1-\sqrt{3}, -1) \cup (-1+\sqrt{3}, +\infty)$

P.TI DI FLESSO $x \in \{-1-\sqrt{3}, -1, -1+\sqrt{3}\}$

$F_1 = (-1-\sqrt{3}, 1-\sqrt{3}) \simeq (-2.7\ -0.7)$

$F_2 = (-1, 1)$

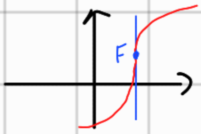
$F_3 = (-1+\sqrt{3}, 1+\sqrt{3}) \simeq (0.7, 2.7)$

b. $$f'(-1) = \frac{-4(-1)^2-8(-1)}{(-1)^2+2(-1)+2} = \frac{-4+8}{1\cancel{-2+2}} = 4$$

la retta tangente alla funzione in $x=1$ sarà del tipo $y = 4x + q$

c. Data $f: D \to \mathbb{R}$, $x_0 \in D$ è detto P.TO DI FLESSO VERTICALE se

$$\lim_{x \to x_0^+} f'(x) = \lim_{x \to x_0^-} f'(x) = \begin{cases} +\infty \\ \text{oppure} \\ -\infty \end{cases}$$

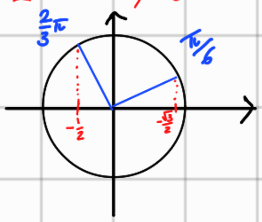


OPPURE

ESERCIZIO 2

a. $$\int \sin(2x)e^{\cos(2x)+\frac{1}{2}}\,dx =$$

$$\left(e^{\cos 2x+1}\right)' = (-2\sin 2x)\,e^{\cos 2x+1}$$



$$= -\frac{1}{2}\int -2\sin(2x)e^{\cos 2x+\frac{1}{2}}\,dx = -\frac{1}{2}e^{\cos 2x+\frac{1}{2}}$$

b. $$\int_{\pi/8}^{\pi/3} \sin 2x\, e^{\cos(2x)+\frac{1}{2}}\,dx = -\frac{1}{2}e^{\cos 2x+\frac{1}{2}}\Big|_{\pi/12}^{\pi/3} = -\frac{1}{2}\left[e^{\cos\left(\frac{2\pi}{3}\right)+\frac{1}{2}} - e^{\cos\left(\frac{\pi}{6}\right)+\frac{1}{2}}\right]$$

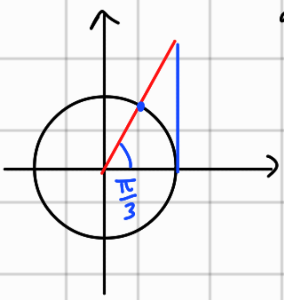
$$= -\frac{1}{2}\left[e^{0} - e^{\frac{\sqrt{3}+1}{2}}\right] = -\frac{1}{2}\left[1 - e^{\frac{\sqrt{3}+1}{2}}\right] = \frac{e^{\frac{\sqrt{3}+1}{2}}-1}{2} \simeq 1.46$$

c. L'integrale al punto b corrisponde alla misura dell'area evidenziata in viola.

ESERCIZIO 3

$$\int_0^{\sqrt{3}} \frac{5t+1}{t^2+1}\,dt = \int_0^{\sqrt{3}} \frac{5t}{t^2+1}\,dx + \int_0^{\sqrt{3}} \frac{1}{t^2+1}\,dt = \left[\frac{5}{2}\ln|t^2+1| + \operatorname{arctg}(t)\right]\Big|_0^{\sqrt{3}}$$

$$= \frac{5}{2}\ln(3+1) + \operatorname{arctg}\sqrt{3} - \underbrace{\ln(1)}_{=0} - \underbrace{\operatorname{arctg}(0)}_{=0} = \frac{5}{2}\ln 4 + \frac{\pi}{3} \simeq 4.51$$



ESERCIZIO 4

X	100	75	50	85	30
Y	175	140	110	170	100

a. $$\bar{x} = \frac{100+75+50+85+30}{5} = \frac{340}{5} = 68$$

Per la mediana riscrivo i dati in ordine crescente 30 50 $\boxed{75}$ 85 100

mediana(x) = 75

$$\sigma_x^2 = \frac{1}{5}\left[(100-68)^2+(75-68)^2+(50-68)^2+(85-68)^2+(30-68)^2\right] =$$

$$= \frac{1}{5}\left(32^2+7^2+(-18)^2+(17)^2+(-38)^2\right) = \frac{3130}{5} = 626$$

Regressione lineare $\quad y = \alpha x + \beta \quad , \alpha = \frac{\sigma_{xy}}{\sigma_x^2} \quad \beta = \bar{y} - \alpha \bar{x}$

$$\bar{y} = \frac{695}{5} = 139$$

$(x_i - \bar{x})$ = 32 7 -18 17 -38

$(y_i - \bar{y})$ = 36 1 -29 31 -39

$$\sigma_{xy} = \frac{1}{5}\left[(32\cdot 36)+(7\cdot 1)+(-18)(-29)+(17\cdot 31)+(-38)(-39)\right]$$

$$= \frac{1}{5}\left[1152+7+522+527+1482\right] = \frac{3690}{5} = 738$$

$$\alpha = \frac{738}{626} = 1.216 \qquad \beta = 139 - \frac{738}{626}68 \simeq 57.03$$

Secondo il modello dopo un allenamento di 60 s i battiti saranno

$$y(60) = \alpha \cdot 60 + \beta = 1.216 \cdot 60 + 57.03 \simeq 130$$